Math 2400 Chapter 3 Notes

A density curve is a curve with the following properties:

1. It is always above the horizontal axis. 2.
2. It has a total area underneath it of 1.

A density curve describes the overall pattern of a distribution.

The area under the curve and above any range of values represents the proportion of all observations that fall in that range.

Where do the mean and median lie on a density curve?

<u>Normal Curves</u> are density curves that are bell-shaped, single-peaked, and symmetric. They describe a special distribution referred to as the Normal Distributions. We describe a specific Normal distribution by giving its mean and standard deviation. We write the normal distribution with mean μ and standard deviation σ as the distribution $N(\mu, \sigma)$.

The 68-95-99.7 Rule In the normal distribution with mean μ and standard deviation σ:

- 68% of the observations fall within σ of the mean .
- 95% of the observations fall within 2σ of the mean .
- 99.7% of the observations fall within 3σ of the mean .

Example: The average full time faculty member in a post-secondary degree-granting institution works an average of 53 hours per week. Assume the distribution is normal with a standard deviation of 2.8 hours and use the 68-95-99.7 rule to answer the following questions.

- What percent of faculty members work between 50.2 hours and 55.8 hours per week?
- What percent of faculty members work more than 58.6 hours per week?
- What percent of faculty members work less than 50.2 hours per week?
- What percent of faculty members work between 50.2 hours and 58.6 hours per week?
- 2.5% of all faculty members work less than how many hours per week?

Example: A manufacturer of automobile batteries claims that the average length of life for its grade A battery is 60 months. However, the guarantee on this brand is for just 36 months. Suppose the distribution of the life of this type of battery is known to be normal with a standard deviation of 10 months. Use the 68-95-99.7 rule to answer the following questions.

- Approximately what percentage of the manufacturer's grade A batteries will last more than 70 months, assuming the manufacturer's claim is true?

- Approximately what percentage of the manufacturer's batteries will last more than 50 months, assuming the manufacturer's claim is true?

- Approximately what percentage of the manufacturer's batteries will last less than 30 months, assuming the manufacturer's claim is true?

- Suppose you purchase one of these batteries and your battery lasts only 27 months. What could you infer about the manufacturer's claim? In a brief sentence, justify your answer.

Standardizing values: As we mentioned earlier, different normal distributions are described by giving their corresponding mean and standard deviation. Comparing values from two different distributions doesn't tell us much unless we consider some sort of standardization between the two distributions.

If x is an observation from a distribution that has mean μ and standard deviation σ , the standardized value of x is given by the formula $z = \dfrac{x-\mu}{\sigma}$. The standardized value is called a z-score.

The z-score tells us how many standard deviation units the value is above or below the mean from its distribution.

Example: A student scored a 65 on a Calculus test that had a mean of 50 and a standard deviation of 10; she scored 30 on a history test with a mean of 25 and a standard deviation of 5. Compare her relative positions on the two tests.

Suppose you take some Normal distribution and standardize every value taken in that distribution. You would get a special distribution called the Standard Normal Distribution. The standard Normal distribution is just the $N(0, 1)$ distribution.

If x has any Normal distribution $N(\mu, \sigma)$ with mean μ and standard deviation σ, then the standardized variable has the standard Normal distribution.

Example: Use Table A to find the proportion of observations from a standard Normal distribution that falls in each of the following regions. In each case, sketch a standard Normal curve and shade the area representing the region.

- $z \leq -2.25$

- $z < -2.25$

- $z > -2.25$

- $z < 1.76$

- $-2.25 < z < 1.76$

Example: Find the value of a standard normal distribution that satisfies each of the following conditions:

- The point z with 25% of the observations falling below it.

- The point z with 40% of the observations falling above it.

Example: In 2000, the scores of men on the math part of the SAT approximately followed a normal distribution with mean 533 and standard deviation 115.

- What proportion of men scored above 500?

- What proportion of men scored between 400 and 600?

Example: Scores on the SAT verbal test in 2002 followed approximately the $N(504, 111)$ distribution. How high must a student score to place in the top 5% of all students taking the SAT?

Example: A marine sales dealer finds that the average price of a previously owned boat is \$6492. He decides to sell boats that will appeal to the middle 66% of the market in terms of price. Find the minimum and maximum prices of the boats that the dealer will sell. The standard deviation is \$1025, and the variable is normally distributed. Would a boat priced at \$5550 be sold in his store?