

Math 3520/6520    Homework 5    October 7, 2009

Due: Tuesday, October 13, 2009

Name: \_\_\_\_\_

Show all work to receive credit!

1. Let  $D$  be the digraph with vertex set  $V(D) = \{u, v, w, z\}$  and Arc set  $A(D) = \{(u, w), (v, u), (v, v), (v, w), (w, z), (z, w)\}$ .

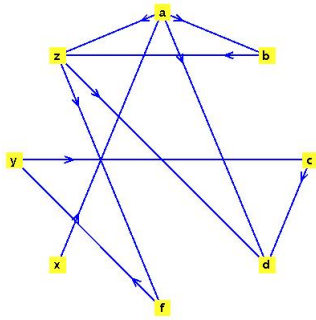
(a) Draw the Digraph  $D$ .

(b) List the indegree of each vertex of  $D$ .

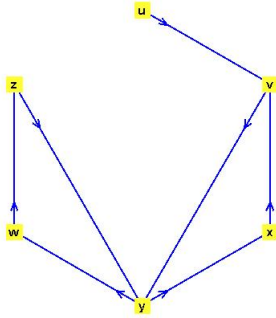
(c) List the outdegree of each vertex of  $D$ .

(d) Draw a subdigraph of  $D$  and give the vertex and edge set of your subdigraph.

2. Find all directed walks from  $x$  to  $d$  in the following digraph:



3. Consider the digraph below and use it to find the following things:



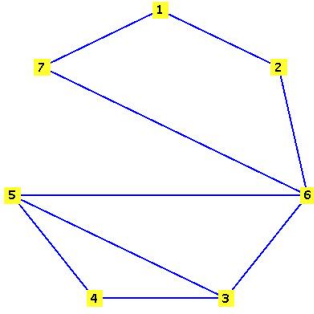
(a) Find a directed walk that is not a directed trail.

(b) Find a directed trail that is not a directed path.

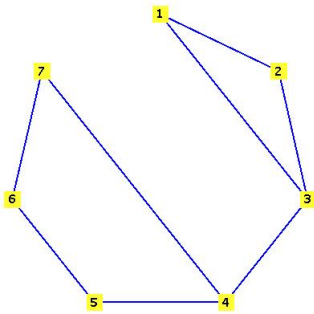
(c) Find a closed directed walk that is not a directed cycle.

4. In a digraph, is it true that a shortest directed walk from a vertex  $x$  to a vertex  $y$  is a directed path from  $x$  to  $y$ ? Briefly explain your answer.

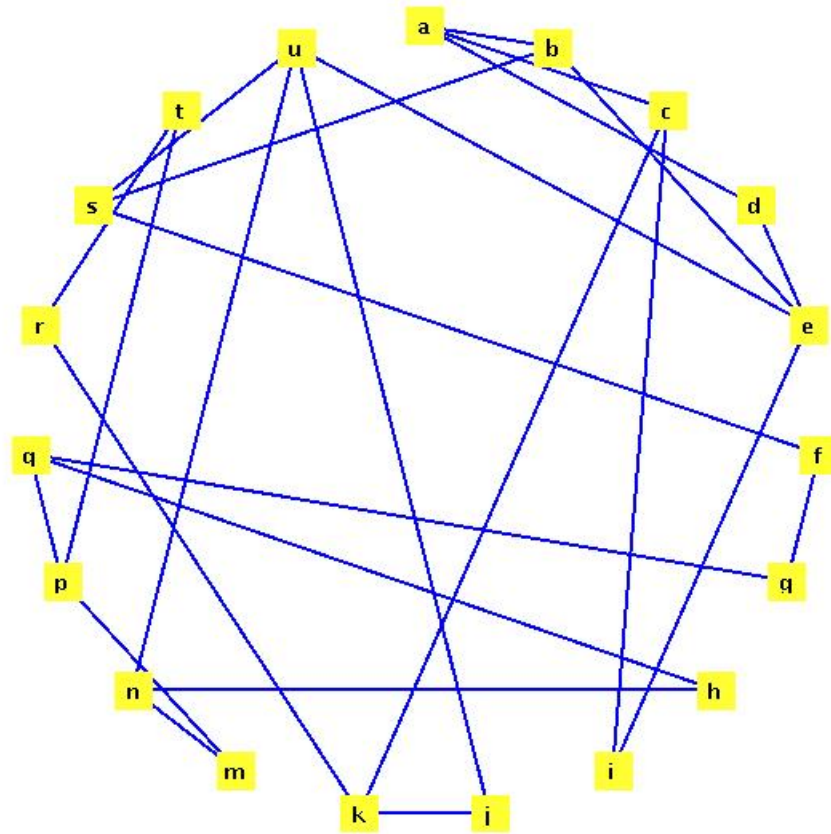
5. For the graph below, determine if the graph is orientable. If it is, then orient the edges appropriately. If it is not, then give a reason why.



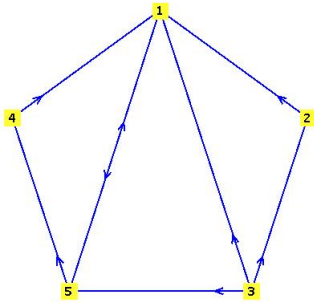
6. For the graph below, determine if the graph is orientable. If it is, then orient the edges appropriately. If it is not, then give a reason why.



7. You are traffic manager for Graphland Valley. Due to traffic congestion caused by a new shopping mall, you have determined that making all streets in town one-way streets will make traffic flow more smoothly. Find an orientation of the edges of the graph below which is the road map of Graphland Valley to obtain a strongly connected digraph so that a good traffic flow will result.



8. Give both the adjacency matrix and the incidence matrix for the digraph given below:



Note that there are two arcs between vertices 1 and 5. One arc goes from 1 to 5 and the other arc goes from 5 to 1. This is the way Maple, by default draws it. Let me know if you have any questions.

9. If  $G$  is a digraph such that every vertex has non-zero outdegree, will it be the case that  $G$  contains a directed cycle? Explain your answer.