Chapter 4 – Differentiation

Motivation: Let's all get on the same page over differentiation rules.

*4.1 – 4.5 read this but I will not formally go over it.

*4.6 Differentiation by Rule

- elementary functions

  $c = \text{constant} \quad c' = 0 \qquad y = x^a \quad y' = ax^{a-1} \qquad y = \sin x \quad y' = \cos x$

  $y = \cos x \quad y' = -\sin x \qquad y = \tan x \quad y' = \sec^2 x \qquad y = e^x \quad y' = e^x$

  $y = \ln x \quad y' = 1/x$

- linear combinations of functions

  $y = au(x) + bv(x) + cw(x) \rightarrow y' = au' + bv' + cw'$

- the product rule

  $y = u(x)v(x) \rightarrow y' = u \cdot v' + v \cdot u'$

  $y = x^2 e^{-ax^2} \rightarrow y' = x^2(-2ax)e^{-ax^2} + e^{-ax^2} \cdot 2x = -2ax^3 e^{-ax^2} + 2xe^{-ax^2}$

- the quotient rule – translate it to a product that is what I always do
- the chain rule

  $y = g(f(x)) \rightarrow y' = \frac{dg}{df} \cdot \frac{df}{dx}$

  $y = \left(1 - e^{-ax}\right)^2 \rightarrow y' = 2\left(1 - e^{-ax}\right) \cdot \left(-e^{-ax}\right) \cdot (-a) = 2ae^{-ax}\left(1 - e^{-ax}\right)$

- inverse functions

  For $y = f(x) \rightarrow x = f^{-1}(y)$ where f^{-1} is the inverse function of f

  Inverse rule: $\frac{dx}{dy} = 1 \Big/ \left(\frac{dy}{dx}\right) \rightarrow \frac{dx}{dy} \bullet \frac{dy}{dx} = 1$

* 4.7 Implicit functions – idea is the same as an inverse function

* 4.8 Logarithmic differentiation

- college algebra

  $\ln(ab) = \ln a + \ln b \qquad \ln\left(\frac{a}{b}\right) = \ln a - \ln b \qquad \ln y^a = a \ln y$

  $\ln\left(e^y\right) = y \qquad e^{\ln y} = y$

- you may have to apply multiple rules (e.g. chain/product rules with logs)

$y = \frac{x^2}{1-3x^4}$ What is the derivative of the $\ln y$?

$$\ln y = \ln\frac{x^2}{1-3x^4} = \ln x^2 - \ln\left(1-3x^4\right) = 2\ln x - \ln\left(1-3x^4\right)$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}\left[2\ln x - \ln\left(1-3x^4\right)\right] = \frac{2}{x} - \frac{1}{1-3x^4}\left(-12x^3\right) = \frac{2}{x} + \frac{12x^3}{1-3x^4}$$

*4.9 Successive differentiation

- you will sometimes need to find higher order derivatives

  $y = x^4 + e^{-ax}$ find the first three derivatives

  $y' = 4x^3 - ae^{-ax}$

  $y'' = 12x^2 + a^2e^{-ax}$

  $y''' = 24x - a^3e^{-ax}$