Chapter 13 Homework Assignment P-Chem Math Fall 2008

Do the following problems at the end of the chapter: 9, 13

9. Confirm that the polynomials $P_4(x)$ & $P_5(x)$ are solutions for the Legendre equation for l = 4 & l = 5, respectively. (*6 pts*)

$$P_4(x) = \frac{1}{8}\left(35x^4 - 30x^2 + 3\right)$$
$$P_4'(x) = \frac{1}{8}\left(35\cdot 4x^3 - 30\cdot 2x\right) = \frac{1}{8}\left(35\cdot 4x^3 - 30\cdot 2x\right) = \frac{1}{8}\left(140x^3 - 60x\right)$$
$$P_4''(x) = \frac{1}{8}\left(420x^2 - 60\right)$$
$$\left(1-x^2\right)y'' - 2xy' + 4(5)y = 0$$
$$\left(1-x^2\right)\frac{1}{8}\left(420x^2 - 60\right) - 2x\frac{1}{8}\left(140x^3 - 60x\right) + \frac{20}{8}\left(35x^4 - 30x^2 + 3\right) = 0$$
$$420x^2 - 60 - 420x^4 + 60x^2 - 280x^4 + 120x^2 + 700x^4 - 600x^2 + 60 = 0$$
$$\left(-420 - 280 + 700\right)x^4 + \left(420 + 60 + 120 - 600\right)x^2 - 60 + 60 = 0$$
$$P_5(x) = \frac{1}{8}\left(63x^5 - 70x^3 + 15x\right)$$
$$P_5'(x) = \frac{1}{8}\left(315x^4 - 210x^2 + 15\right)$$
$$P_5''(x) = \frac{1}{8}\left(1260x^3 - 420x\right)$$
$$\left(1-x^2\right)y'' - 2xy' + 30y = 0$$
$$\left(1-x^2\right)\left(1260x^3 - 420x\right) - 2x\left(315x^4 - 210x^2 + 15\right) + 30\left(63x^5 - 70x^3 + 15x\right) = 0$$
$$1260x^3 - 420x - 1260x^5 + 420x^3 - 630x^5 + 420x^3 - 30x + 1890x^5 - 2100x^3 + 450x = 0$$
$$\left(-1260 - 630 + 1890\right)x^5 + \left(1260 + 420 + 420 - 2100\right)x^3 + \left(-420 - 30 + 450\right)x = 0$$

13. Use the recurrence relation (13.33) to find $H_6(x)$.

$$H_{n+1}(x) - 2xH_n(x) + 2nH_{n-1}(x) = 0 \rightarrow H_n(x) = \frac{H_{n+1}(x)}{2x} + \frac{2nH_{n-1}(x)}{2x}$$
$$H_7(x) = 128x^7 - 1344x^5 + 3360x^3 - 1680x$$
$$\frac{H_7(x)}{2x} = 64x^6 - 672x^4 + 1680x^2 - 840$$
$$\frac{2nH_5(x)}{2x} = \frac{2\cdot 6}{2}\frac{32x^5 - 160x^3 + 120x}{x} = 192x^4 - 960x^2 + 720$$
$$H_6(x) = 64x^6 + \left(192 - 672\right)x^4 + \left(1680 - 960\right)x^2 + 720 - 840 = 64x^6 - 480x^4 + 720x^2 - 120$$